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Theoretical groundwork: An extension to the double Hertz model for adhesion between elastic cylinders

N.H.M. Zini[1,2,*], M.B. de Rooij[1], N. Ismail[1,2], D.J. Schipper[1], A. Akchurin[1]

[1)] Department of Surface Technology and Tribology, Faculty of Engineering Technology,
University of Twente, the Netherlands.
[2)] Faculty of Mechanical Engineering, Universiti Teknikal Malaysia Melaka,
Hang Tuah Jaya, 76100 Durian Tunggal, Melaka, Malaysia.

[*]Corresponding e-mail: n.hilwabintimohdzini@utwente.nl

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ABSTRACT – The theoretical groundwork of an alternative approach to the adhesive contact problem of elastic cylinders is proposed, extending the work previously done on the double-Hertz model for circular and line contacts. A cohesive zone model with general formulations for elliptical contact between elastic cylinders is currently developed. By nature, this approach will be suitable in the case involving the bodies of general shape. The advantage of this model is that the analysis depends only on established results of the Hertz model. Initial results will be presented and discussed.

1. INTRODUCTION

It has been observed that surface adhesion plays a central role in the mechanics of surface contacts in particular at small scale. So far, there is only one established adhesive contact model to describe the mechanics of elliptical contact; the Johnson-Kendall-Roberts (JKR) model. However, JKR elliptical model is shown to become inaccurate when the skew angles between the two cylinders are small and the resulting contact is highly eccentric, as reported from previous result [1]. A significant characteristic of JKR model is that it does not consider the adhesion forces outside the contact area and therefore suitable only for soft materials with relatively large Tabor parameters [2].

The present paper is aimed to extend the application of double-Hertz model to an elliptical contact. As a cohesive zone based solution, the model developed should be able to describe the effect of surface adhesion for interacting materials with general properties.

2. DOUBLE-HERTZ MODEL

Without the effect of adhesion, the Hertz type solutions for the general cases are given by Equations (1) to (6) [3]. Figure 1 shows an example of such cases where two cylinders crossed each other with skew angle, θ between 0° to 90°. It is assumed that the shape of contact area is elliptical, with semi-axes *a* and *b*.

Pressure distribution:

$$p(x,y) = p_0\{1-(x/a)^2-(y/b)^2\}^{1/2} \tag{1}$$

Which act over the area bounded by ellipse

$$(x/a)^2+(y/b)^2-1=0 \tag{2}$$

Total load acting on the ellipse:

$$P = 2\pi abp_0/3 \tag{3}$$

Surface displacements:

For $r \leq a$

$$w(r) = \frac{p_0 b}{E^*}\left[K(e) - \frac{\{K(e)-E(e)\}}{e^2a^2}x^2 - \left\{\frac{E(e)}{e^2b^2} - \frac{K(e)}{e^2a^2}\right\}y^2\right] \tag{4a}$$

For $r \geq a$

$$w(r) = \frac{p_0 ab}{2E^*}\int_s^\infty \left(1-\frac{x^2}{a^2+w} - \frac{y^2}{b^2+w}\right)\frac{dw}{\{(a^2+w)(b^2+w)w\}^{1/2}} \tag{4b}$$

Where *E(e)* and *K(e)* are complete integrals of argument: $e = (1-b^2/a^2)^{1/2}$
(5)

With $b < a$, P is the load, p_0 is the maximum contact pressure, E^* is the contact modulus and *s* is the positive root of equation:

$$x^2/(a^2+s) + y^2/(b^2+s) = 1. \tag{6}$$

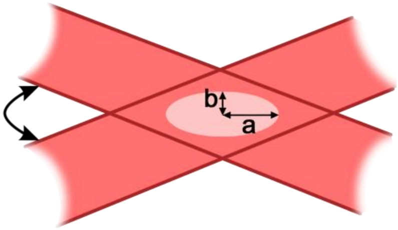

Figure 1 Crossed cylinders with skew angle, θ between 0° to 90° produced an elliptical contact with semi major axis *a* and semi minor axis *b*.

Adhesion modeling relies on the combination of adhesive tensile stresses outside the contact area with compressive stresses inside, while ensuring that the contacted bodies still conform within the contact area boundary. The idea initiated from the Derjaguin model

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where the deformation resulted from the adhesive stresses were neglected, so that the Hertzian internal pressures can be retained [4].

The basis of double-Hertz model is that the adhesive tensile stresses is represented by the difference of two Hertzian pressure distributions of semi major axis a and semi major axis c, with $c > a$ given by:

For $r \leq a$

$$p(x,y) = p_0\left[\{1-(x/c)^2-(y/d)^2\}^{1/2} - \{1-(x/a)^2-(y/b)^2\}^{1/2}\right] \tag{7a}$$

For $a \leq r \leq c$

$$p(x,y) = p_0\{1-(x/c)^2-(y/d)^2\}^{1/2} \tag{7b}$$

The displacement produced from the difference between two Hertzian solutions:

For $r \leq a$

$$w(r) = \frac{p_0}{E^*}\left[b\left(K(e) - \frac{\{K(e)-E(e)\}}{e^2a^2}x^2 - \left\{\frac{E(e)}{e^2b^2} - \frac{K(e)}{e^2a^2}\right\}y^2\right) - d\left(K(e) - \frac{\{K(e)-E(e)\}}{e^2c^2}x^2 - \left\{\frac{E(e)}{e^2d^2} - \frac{K(e)}{e^2c^2}\right\}y^2\right)\right] \tag{8a}$$

For $a \leq r \leq c$

$$w(r) = \frac{p_0}{E^*}\left[\left(\frac{ab}{2}\int_s^\infty\left(1 - \frac{x^2}{a^2+w} - \frac{y^2}{b^2+w}\right)\frac{dw}{\{(a^2+w)(b^2+w)w\}^{1/2}}\right) - d\left(K(e) - \frac{\{K(e)-E(e)\}}{e^2c^2}x^2 - \left\{\frac{E(e)}{e^2d^2} - \frac{K(e)}{e^2c^2}\right\}y^2\right)\right] \tag{8b}$$

3. PRELIMINARY RESULT

Figure 2 shows the distributions of the normalized pressures p/p_0, obtained from the difference between two Hertzian solutions. The pressure distribution over $a \leq r \leq c$ is shown to decline at a steady rate from the maximum value at $r = a$ to zero at $r = c$. Equation (7b) is use to model the adhesive tensile traction over $a \leq r \leq c$, which produces the final surface traction distribution.

It is observed in Figure 3 that higher displacement difference is produced in region $a \leq r \leq c$ due to the presence of adhesive tensile traction which results in the increase of the original contact area. For the region of $r \leq a$, the current model exhibits a nearly constant displacement with gradual decrease compared to the circular double-Hertz model where uniform displacement is produced within the boundary.

4. CONCLUSION

A new model of adhesion due to surface forces for elliptical contact has been developed. The proposed model can serve as an alternative solution for adhesive elliptical contact and applicable to a wide range of contact conditions.

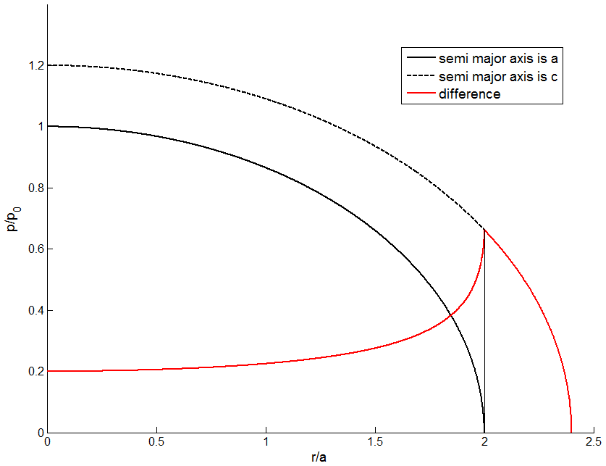

Figure 2 The difference between two Hertzian solutions with semi major axes a and c for surface pressure.

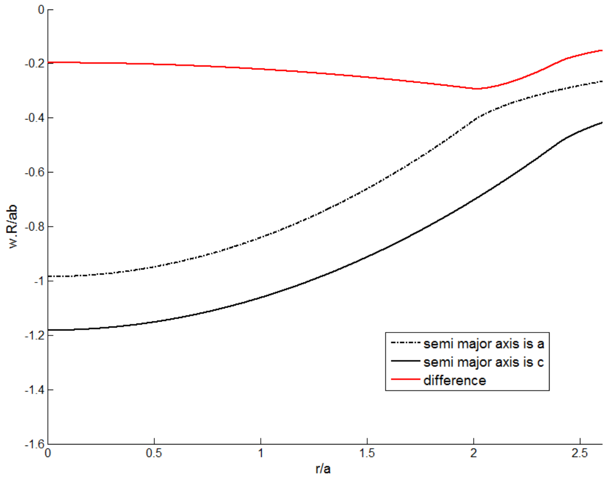

Figure 3 Corresponding surface displacement.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

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